Audio and Video Digital Signal Conversion by Elliptic Filter and MATLAB Software

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***Abstract*— In this study, towards analog and digital elliptic filters, use of transfer function through the choice of a particular filter structure from a set of design specifications to achieve the step change in transfer function analysis. Elliptic filter design with two optimization criteria (1) maximally flat amplitude response and (2) small amplitude ripple response, and the excellent performance of the filter available. There are many different filter design methods discussed in this paper is the first for the analog method, used in analog filters. The specified frequency response with amplitude response and phase response which obtained through the rational transfer function approximation. In addition to the order of the filter for different analysis, the use of continuous signals into discrete Z-transform system by using (1) Pulse constant conversion method (2) Bilinear transformation method two conversion methods. This paper and using MATLAB software to calculate the analysis results, and the results plotted graphically confirmed that the two algorithms are calculated from the Z-transform is very similar to results with the correct solution, using MATLAB software is used to evaluate the convenience, and By the results of its analysis to support the correctness of the conversion result, and confirmed that the bilinear transformation method than the pulse of the same conversion method has a better conversion results.**

***Keywords*— Audio and Video, Elliptic filter, Signal processing, Matlab, Bilinear transformation method, Impulse invariance method**

I. Introduction

Elliptic filter widely used in the realization of speech processing. Because in speech analysis and speech synthesis that has the excellent processing power ability, the mathematical model of analysis by a transfer function can be described, or you can use the amplitude response, phase response described. Through literature Ma et al. [1] applied to the circuit signal analysis, Romero et al. [2] engaged in the ladder filter of the test, Gordon et al. [3] the reduction in filter design and analysis of their response, Alarcon et al. [4] the use of three-pole Butterworth analog filter for ECG signal of the research. The study of algorithms [5], first to use the Laplace transform, from the regularization of the classical analog low-pass filter. For the pre-distorting the cutoff frequency, so the other frequency selective filter is also applicable, such as high-pass, band pass and band stop filter. For the frequency conversion of analog design with a diameter cut-off frequency at different degrees of order low-pass Butterworth filter is normalized Butterworth low-pass filter start and uses the filter cutoff frequency and order of replacement which is s functional. This is an example of frequency conversion. Using the same method can be normalized low-pass filter, converted to other types of frequency selective filters. Finally, the output of the system to produce a continuous time signal reconstruction filter, on behalf of the original input signal is filtered results. This paper and using MATLAB software to analyze the results of calculations [6], and the results graphically plotted for reference comparison. Since elliptical filters better than the Butterworth filter's performance, so this article gives special seminars. Recently a more linear elliptic filter, designed to be more flexible and has a wide bandwidth [7,8], on the other filter current transmission control is also a new development [9], and have very low band-pass sensitivity and compensation effect [10], usually require high-order elliptical filter before they can achieve the above effect [11-14]. Through references [1-14] and to consolidate research and discussion to make a design of high-order elliptic filter is one of the points this study, this study first use of the Laplace transform algorithm, followed by its digital technology, making it the digital filter and use the calculation (computation) to perform continuous-time signal filtering action. Design frequency-selective filter, is used to convert the continuous-time signal corresponding to the series. Digital filter in a series of sampling points on the basis of processing sequence to generate new series,

new series followed by a digital to analog (D / A) converter, conversion to the corresponding continuous time signal. Finally, the output of the system to produce a continuous time signal reconstruction filter, on behalf of the original input signal is filtered results. In this study using Impulse invariance method and Bilinear transformation method to deal with the digital signal transfer, change of continuous signals into discrete Z-transform system.

II. MAIN CONTAINS

Signal processing algorithms often occur in a series of complex operations can easily make the calculation error, re-verification must spend a lot of time in order to calculate the convenience and accuracy of using MATLAB to write program makes saving a lot of time on operations, and therefore Our main areas of this monograph is often used in engineering to math up development, such as; linear algebra, polynomial solving, probability calculations ... etc., developed the program features allow users more convenient operation. Elliptic filter is sometimes called Cauer filter. This filter has the same ripple for band pass and band stop. Discussed in the previous filter in the known order of the filter, band pass and stop with error, Elliptic filter with minimal transient bandwidth. The size of the square of the response is

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 U_N(\Omega/\Omega_c)} \quad (1)$$

Where $U_N(x)$ is the N order elliptic function, while ε is the pass band ripple of the constant.

2-1 Frequency transformation

There are four basic types of frequency selective filters, for low pass, high pass, band pass and band stop. Discussed in the previous design skills, we only take into account the low-pass filter. Low-pass filter can be regarded as the prototype filter and the system functions fairly easy to obtain. Furthermore, to design high-pass or band-pass or band stop filter, frequency conversion can be used easily to obtain. Frequency conversion can be done in two ways. In the analog frequency conversion, the analog prototype filter $H_P(s)$ into a desired system function of the digital filter of another analog system function $H(s)$. Then use the corresponding skills, into a system function $H(z)$ of the digital filter. In the digital frequency conversion, the analog prototype filters into a first digital domain, with the system function $H_P(z)$. Then use of frequency conversion, so that can be converted into the digital filter.

2-2 Analog frequency transformation

Analog frequency conversion formula used in a prototype low-pass filter into a low-pass (with a different cut-off frequency), high pass, band pass or band stop filter, as shown below.

(i) low-pass filter cut-off frequency Ω_C to a new low pass filters cut-off frequency Ω_C*:

$$s \to \frac{\Omega c}{\Omega_C^*} \quad (2)$$

Therefore, if the prototype filter response of the system is $H_P(s)$, the low-pass filter of the system response will

$$H(s) = H_P\left(\frac{\Omega c}{\Omega_C^*} s\right) \quad (3)$$

(ii) low-pass filter cut-off frequency Ω_C to a new high-pass filters cut-off frequency Ω_C*:

$$s \to \frac{\Omega_C \Omega_C^*}{s} \quad (4)$$

the system function of high-pass filter is

$$H(s) = H_P\left(\frac{\Omega c \Omega_C^*}{s}\right)$$

(5)

(iii) low-pass filter cut-off frequency Ω_C to the cut-off frequency Ω_1 and high cut-off frequency of the band pass filter Ω_2 :

$$s \to \Omega_C \frac{s^2 + \Omega_1 \Omega_2}{s(\Omega_1 - \Omega_2)}$$

(6)

the system function of band-pass filter is

$$H(s) = H_p\left(\Omega_C \frac{s^2 + \Omega_1 \Omega_2}{s(\Omega_2 - \Omega_1)}\right)$$

(7)

(iv) cutoff frequency Ω_C for low pass filter cutoff frequency to the low Ω_1 and high cutoff frequency Ω_2 of the band stop filter:

$$s \to \Omega_C \frac{s(\Omega_2 - \Omega_1)}{s^2 + \Omega_1 \Omega_2}$$

(8)

the system function of band-stop filter is

$$H(s) = H_p\left(\Omega_C \frac{s(\Omega_2 - \Omega_1)}{s^2 + \Omega_1 \Omega_2}\right)$$

(9)

2-3 Digital frequency conversion

As in the analog domain, frequency conversion in the digital domain is feasible. In the digital domain into the frequency converter by replacing the function Z^{-1} to achieve, that is, $f(Z^{-1})$. This corresponds to take into account the stability criteria. All poles must fall within the unit circle corresponds to its own, and also corresponds to the unit circle itself. To achieve the corresponding unit circle to itself, for r = 1, Z = r * e^{jw} .

$$e^{-j\omega} = f(e^{-j\omega}) = \left|f(e^{-j\omega})\right| e^{j\arg[f(e^{-j\omega})]}$$

(10)

Analog frequency transformation formulas as shown in Table (1)

Therefore , to the all frequency , must be $\left|f(e^{-j\omega})\right| = 1$ 。 So that, the correspondence all pass filter can be written as

$$f(z^{-1}) = \pm \prod_{k=1}^{n} \frac{z^{-1} - \alpha_k}{1 - \alpha_k z^{-1}}$$

(11)

Table (1) Analog frequency transformation formulas

Analog	Transformation
Low pass	$s \to \frac{\Omega c}{\Omega_C^*} s$
High pass	$s \to \frac{\Omega_C \Omega_C^*}{s}$
Band pass	$s \to \left(\Omega_C \frac{s^2 + \Omega_1 \Omega_2}{s(\Omega_2 - \Omega_1)}\right)$
Band stop	$s \to \Omega_C \frac{s(\Omega_2 - \Omega_1)}{s^2 + \Omega_1 \Omega_2}$

In order to stabilize the prototype filter to be stable filter must set $|\alpha k| < 1$. For the conversion prototype low-pass digital filter to the digital low-pass, high-pass, band-pass, or band-stop filter, the conversion formula from (10) obtained, as shown in Table (2).

TABLE (2) CONVERSION PROTOTYPE LOW-PASS DIGITAL FILTER TO THE DIGITAL LOW-PASS, HIGH-PASS, BAND-PASS, OR BAND-STOP FILTER

Types	Transformation	Design parameters
Low-pass	$z^{-1} \to \frac{z^{-1} - a}{1 - az^{-1}}$	$a = \frac{\sin[(\omega_c - \omega_c^*)/2]}{\sin[(\omega_c + \omega_c^*)/2]}$
High-pass	$z^{-1} \to -\frac{z^{-1} + a}{1 + az^{-1}}$	$a = -\frac{\cos[(\omega_c - \omega_c^*)/2]}{\cos[(\omega_c + \omega_c^*)/2]}$

Band-pass	$z^{-1} \to \frac{z^{-2}-a_1 z^{-1}+a_2}{a_2 z^{-2}-a_1 z^{-1}+1}$	$a_1 = -2\alpha K/(K+1)$ $a_2 = (K-1)/(K+1)$ $\alpha = \frac{\cos[(\omega_2+\omega_1)/2]}{\cos[(\omega_c-\omega_1)/2]}$ $K = \cot\left(\frac{\omega_2-\omega_1}{2}\right)\tan\left(\frac{\omega_2}{2}\right)$
Band-stop	$z^{-1} \to \frac{z^{-2}-a_1 z^{-1}+a_2}{a_2 z^{-2}-a_1 z^{-1}+1}$	$a_1 = -2\alpha/(K+1)$ $a_2 = (1-K)/(1+K)$ $a = \frac{\cos[(\omega_2+\omega_1)/2]}{\cos[(\omega_2-\omega_1)/2]}$ $K = \tan\left(\frac{\omega_2-\omega_1}{2}\right)\tan\left(\frac{\omega_c}{2}\right)$

Frequency conversion can be done either two available techniques, however, important to note that what techniques to use. Move out, the pulse invariant transformation is not suitable for high frequency high-pass resonant or band-pass filter. In this case, assuming a low-pass prototype filter which uses the analog frequency conversion into high-pass filter, and then using impulse-invariance techniques into the digital filter. This will cause aliasing problems. However, if the same low-pass filter prototype, the first use of pulse techniques to convert non-variant digital filter, then after using the digital frequency conversion into high-pass filter, phase will not have any aliasing problem. Regardless of when to use the bilinear transformation, whether using analog or digital frequency conversion frequency converter is not important. In this case, analog and digital frequency conversion techniques will get the same results.

III. MATLAB EXAMPLES

Example 1**:** In accordance with the following specifications makes analog elliptical low-pass filter design. Wp = 0.2π; Ws = 0.3π; Rp = 1dB; As = 16dB;

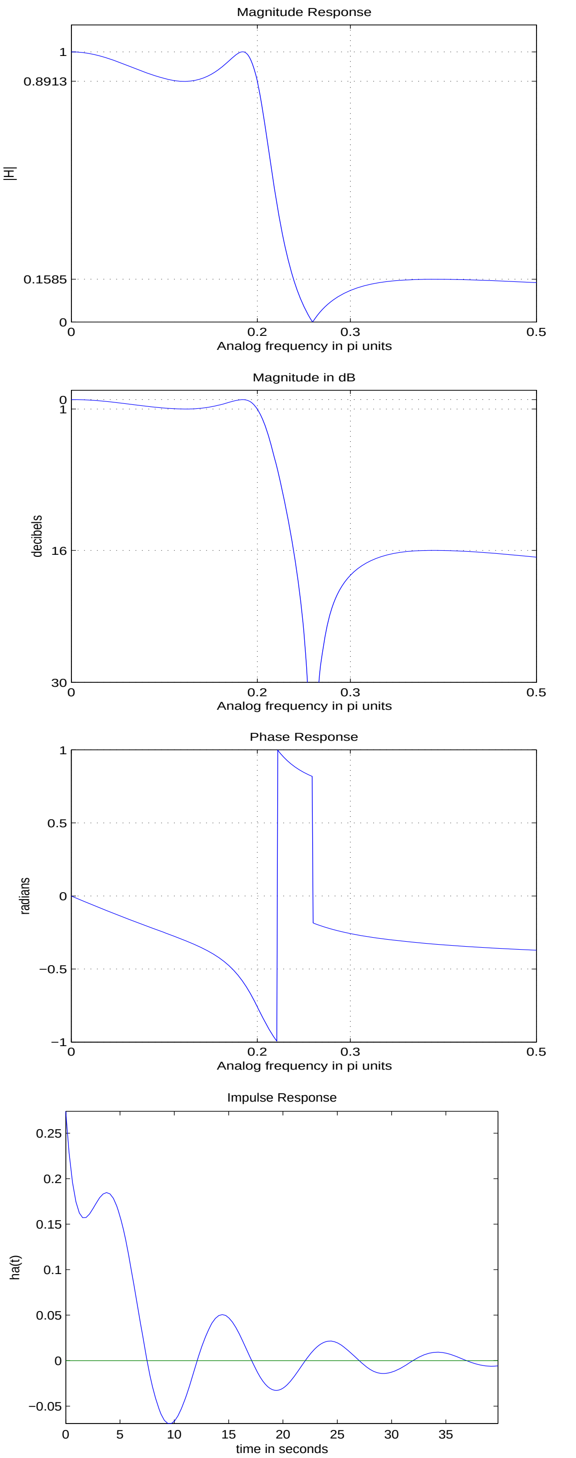
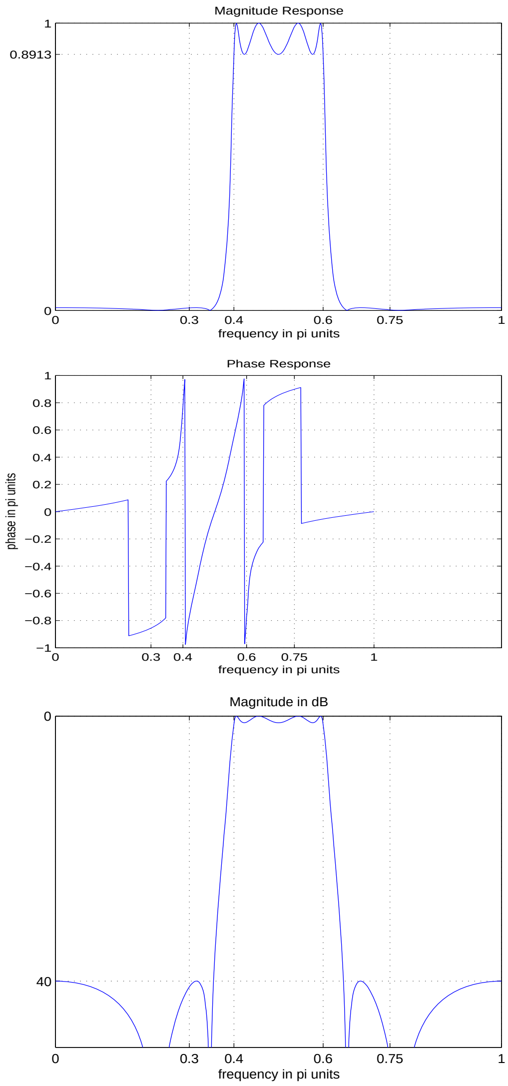


Figure 1. The relationship for analog elliptical low-pass filter frequency to amplitude and phase

From above Example 1 we can find out that analog elliptical low-pass filter at the confine specifications Wp = 0.2π; Ws = 0.3π; Rp = 1dB; As = 16dB will produce a small ripples at the low pass region and obtain a clearly

slope for the next region, it is a good transform results for a low pass elliptic filter.

Example 2: In accordance with the following specifications makes digital elliptical low-pass filter design. Wp = [0.4π 0.6π]; Ws = [0.3π 0.75π] ; Rp = 1dB; As = 40dB;



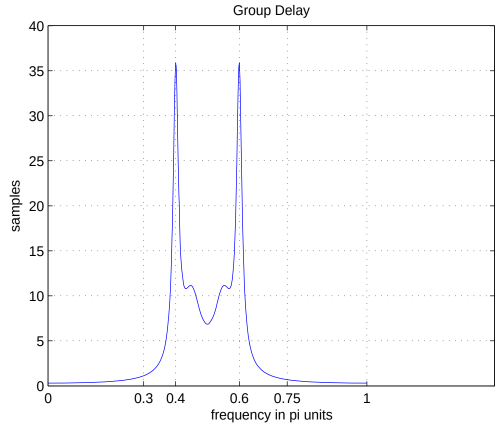


Figure 2. The relationship for digital elliptical low-pass filter frequency to amplitude and phase

From above Example 2 we can find out that analog elliptical band-pass filter at the confine specifications Wp = [0.4π 0.6π]; Ws = [0.3π 0.75π] ; Rp = 1dB; As = 40dB will produce a small ripples at the band pass top region and obtain a clearly slope for the other regions, it is a also good transform results for a band pass elliptic filter.

Example 3: In accordance with the following specifications makes digital elliptical low-pass filter design. Wp = 0.3; Ws = 0.4; Rp = 1dB; As = 20dB;

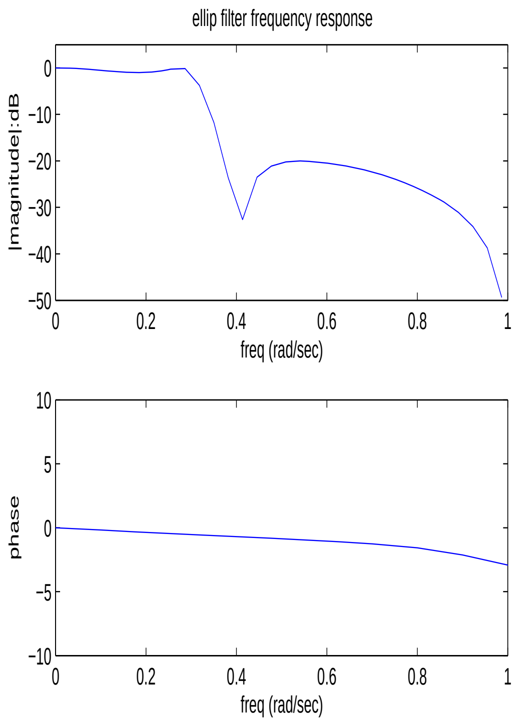


Figure 3-a. The relationship for digital elliptical low-pass filter frequency to amplitude and phase

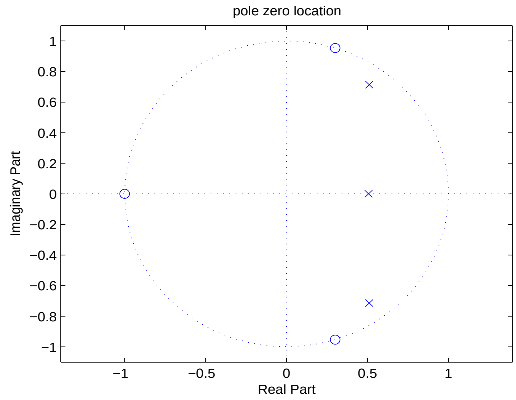


Figure 3-b. Digital elliptical low-pass filter pole and zero points map

From above Example 3 we can find out that digital elliptical low-pass filter at the confine specifications Wp = 0.3; Ws = 0.4; Rp = 1dB; As = 20dB will produce a small ripples at the low pass region and obtain a clearly slope for the next region, it is a good transform results for a low pass elliptic filter. On the other hand, from Figure 3-b has shown that all the pole and zero points are convergence at the unit circle.

Example 4: In accordance with the following specifications makes digital elliptical high-pass filter design. Wp = 0.4; Ws = 0.3 ; Rp = 1dB; As = 20dB;

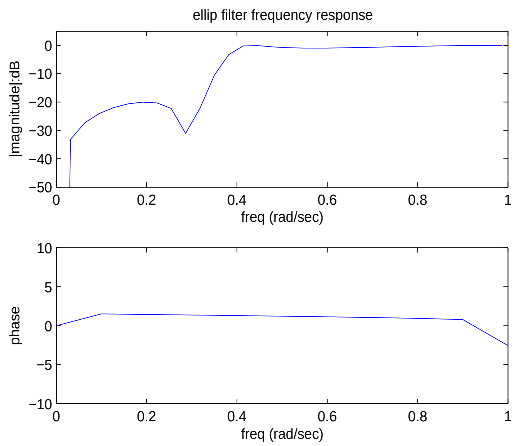


Figure 4-a. The relationship for digital elliptical high-pass filter frequency to amplitude and phase

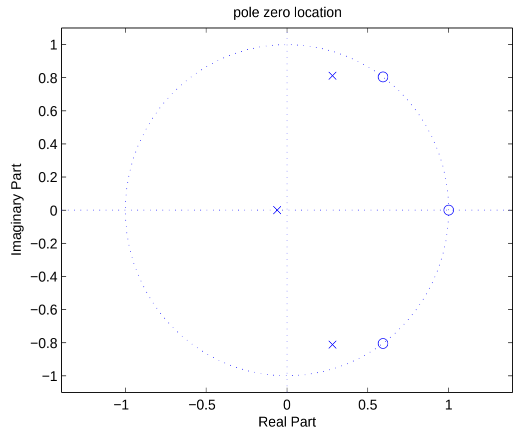


Figure 4-b. Digital elliptical high-pass filter pole and zero points map

From above Example 4 we can find out that digital elliptical high-pass filter at the confine specifications Wp = 0.4; Ws = 0.3 ; Rp = 1dB; As = 20dB will produce a small ripples at the high pass region and obtain a clearly slope for the next region, it is a good transform results for a high pass elliptic filter. On the other hand, from Figure 4-b has shown that all the pole and zero points are convergence at the unit circle.

Example 5: In accordance with the following specifications makes digital elliptical Band-pass filter design. Wp = [0.3 0.5]; Ws = [0.2 0.55] ; Rp = 1dB; As = 20dB;

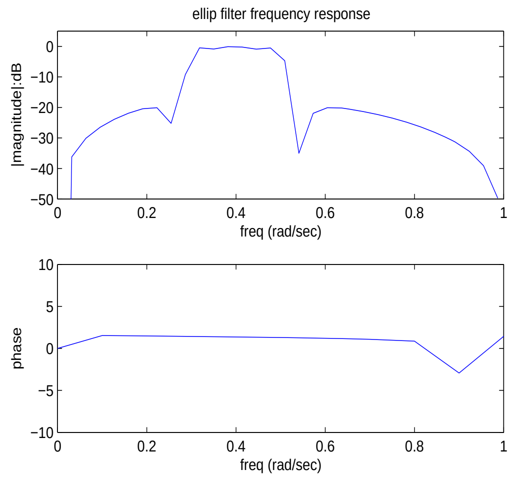


Figure 5-a. The relationship for digital elliptical band-pass filter frequency to amplitude and phase

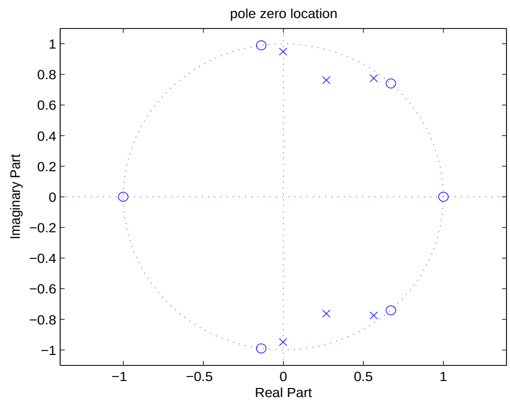


Figure 5-b. Digital elliptical band-pass filter pole and zero points map

From above Example 5 we can find out that digital elliptical band-pass filter at the confine specifications Wp = [0.3 0.5]; Ws = [0.2 0.55] ; Rp = 1dB; As = 20dB will produce a small ripples at the band pass region and obtain a clearly slope for the next region, it is a good transform results for a band pass elliptic filter. On the other hand, from Figure 4-b was shown that all of the pole and zero points are convergence at the unit circle.

Example 6: In accordance with the following specifications makes digital elliptical Band-stop filter design. Wp = [0.2 0.55]; Ws = [0.3 0.5] ; Rp = 1dB; As = 20dB;

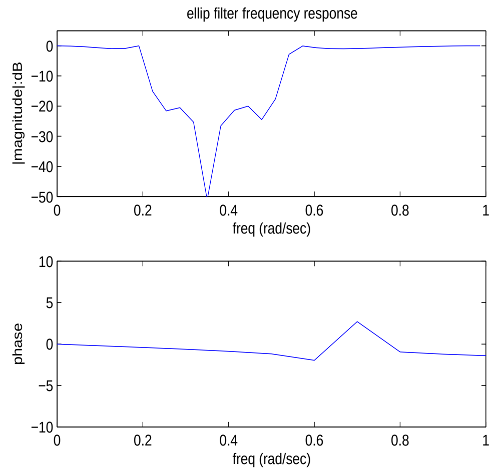


Figure 6-a. The relationship for digital elliptical band-stop filter frequency to amplitude and phase

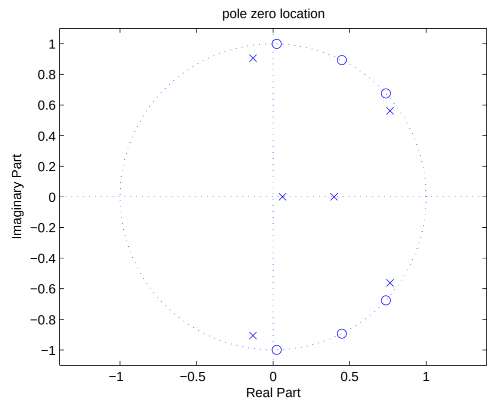


Figure 6-b. Digital elliptical band-stop filter pole and zero points map

From above Example 6 we can find out that digital elliptical band-stop filter at the confine specifications produce a good transform results for elliptic filter. The most important thing is finding that the bilinear transformation method is better than the Impulse-invariance method for the high pass filter status.

Example 7. Impulse-invariance method and bilinear transformation method will be converted analog elliptical low-pass filter to high-pass digital filter. Filter specification are

$$\omega_{pL} = 0.2\pi, \omega_{pH} = 0.2\pi, R_p = 1\text{dB}$$

$$\omega_s = 0.3\pi, A_s = 15\text{dB}$$

ω_{pL} :Low-pass digital frequency

ω_{pH} : High-pass digital frequency

R_p : Ripple of band-pass

ω_s : Band-stop digital frequency

A_s : Band-stop digital frequency decay

This article by the above theoretical analysis, first create a simulation system by Laplace transform method to expand as part of the fractions, and using MATLAB software to analyze the selected time interval of its calculation and the results plotted graphically in Figure 1 . Impulse-invariant method and Bilinear transformation method will be converted to analog elliptical low-pass filter

and high-pass digital filter.

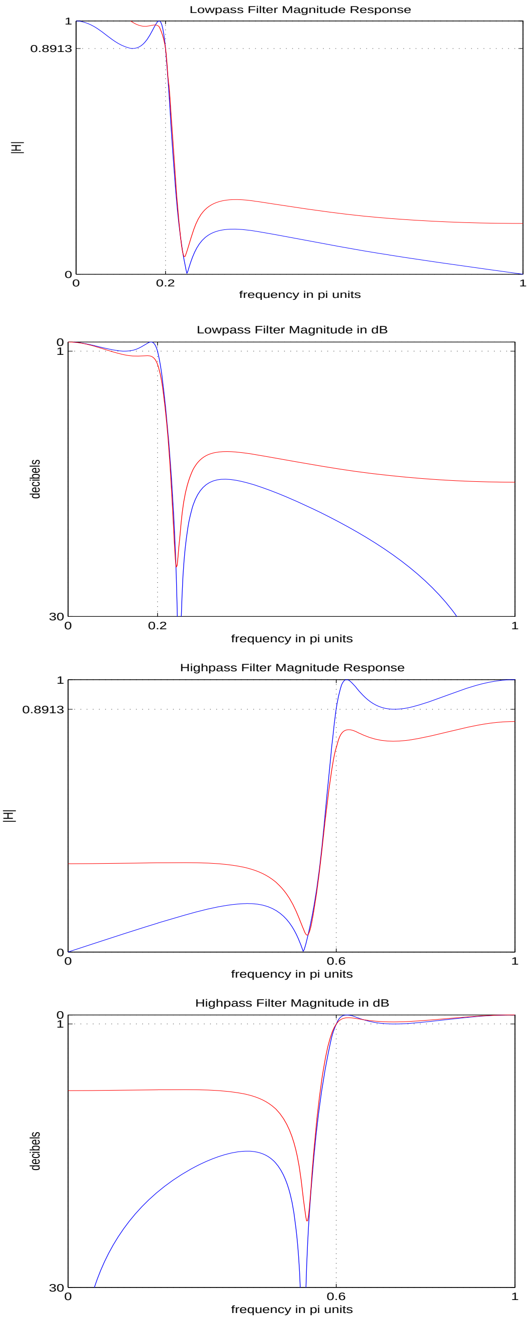


Figure 7 Using Impulse-invariance transformation method and bilinear elliptic low-pass filter method to high-pass filter for the frequency and amplitude relationship

From above Example 7 we can find out that digital elliptical filter using Impulse-invariance transformation method and bilinear elliptic low-pass filter method to high-pass filter at the confine specifications Wp = [0.2 0.55]; Ws = [0.3 0.5] ; Rp = 1dB; As = 20dB; will produce a small ripples beside the band stop regions and obtain a clearly slope for the next region, it is a good transform results for a band stop elliptic filter.

IV. Results and Discussions

This article by the above theoretical analysis, first by the analog frequency conversion from the regularization of the classical analog low-pass filter transfer function $H_a(s)$ start, pre-distorting the cut-off frequency, and then apply the bilinear transformation equation to produce the equivalent digital filter H(z). For the frequency conversion of analog design with a diameter-degree low-pass cut-off frequency in the elliptic filter is normalized elliptic low-pass filter Ω_0 from the start, and with replacement of s/Ω_0. Using the same method can be normalized low-pass filter, converted to other types of frequency selective filters, such as high-pass, band-pass, and band-stop. This series of computer simulation using MATLAB software for analysis of related functions and a graphical plot of the conversion result, as shown in Figure 1-6. Finally, there is the discrimination, by (a) the same impulse-invariance method (b) Bilinear transformation method, Laplace equation can be converted to the Z-transform and discrete systems using MATLAB software to analyse the selected time interval of its calculation and the results plotted graphically in Figure 7, the resulting two kinds of digital signal conversion of a graphic comparison of the results are very similar. This calculation by the simulation system using MATLAB software for analysis of related functions and the results plotted graphically, and to get good simulation results. Found in Figure 3 of the high-pass digital filter of the amplitude of the Bilinear transformation by Impulse-Invariance produce more amplitude than higher-up about 25%.

V. CONCLUSIONS

(1) This analog elliptic filter system, through the establishment of model. Analog signals with the proposed digital signal equation, obtained using MATLAB analysis and calculation of the order of the impulse response of different situations, and once again confirmed that the number of higher order than those of their turning point clear.

(2) for discrete processing and digital filter system to get, by (1) Pulse constant conversion method (2) Bilinear transformation method and other two conversion methods, Laplace equation can be converted to use the Z- conversion of discrete digital system, and selected using MATLAB software to analyze its time interval is calculated and obtained good simulation results, and bilinear transformation method to get the same conversion than the pulse method has a better conversion results.

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